

# Evaluating Real Estate Development Using Real Options Analysis

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## Abstract

This paper describes a simple method for using real options analysis to evaluate commercial real estate projects when development can be suspended. The approach described can be applied to projects at any stage of construction, which is especially useful when market conditions are poor and suspension of many partly-completed projects is being considered. All the calculations can be performed in a spreadsheet and only one parameter—the volatility of the price of a completed project—needs to be estimated in addition to those required for static DCF analysis.

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## 1 Introduction

Static DCF analysis remains the valuation tool of choice for many practitioners, despite its failure to incorporate the effects of managerial flexibility. This flexibility takes many forms, from the ability to delay beginning construction of a planned project to complicated sequencing of large developments that may span several distinct stages spread over many years. Real options analysis offers the potential to value projects in a way that recognizes the existence of the flexibility available to developers and, moreover, to determine the best way to exercise this flexibility. In contrast, static DCF analysis implicitly assumes that the entire sequence of decisions regarding completion of a development is made before construction actually begins and, no matter what might happen afterwards, is adhered to strictly. The main advantages of static DCF analysis are that it is simple to implement and, because it is implemented in a fairly systematic way, the results are easily communicated. In contrast, real options analysis is often

implemented using mathematical tools unavailable to many practitioners. Furthermore, many different approaches are possible, and no single approach has emerged as “standard practice”.

Real options analysis has made substantial inroads into the academic study of real estate markets. For example, it has been used to analyze the value of vacant land (Titman, 1985) and the flexibility embedded in typical leases (Grenadier, 1995); there is a large (and growing) literature using real options analysis to analyze the equilibrium behavior of property prices (Grenadier, 1996; Guthrie, 2009a). However, progress on implementing real options analysis in practice has been much slower. While various case studies have appeared in the practitioner literature, there seems to be no widely-used systematic approach to real options analysis applied to real estate problems.

Several recent books attempt to make the insights of real options analysis available to a wider range of practitioners. This paper explains how the approach presented in one of them, Guthrie (2009b), can be applied to a wide variety of real estate applications. It avoids the differential equations and complicated simulations that are often used to carry out real options analysis, instead using the familiar binomial-tree approach. The key step, and what distinguishes it from standard derivative pricing situations, is the identification of a small number of discrete stages of development and the allocation of the various items of capital expenditure amongst these stages. This allocation reduces project evaluation to the analysis of a reasonably straightforward compound option pricing problem. Construction lags add a slight complication to the analysis, but they are easily handled in a systematic fashion.

The data requirements for this approach are not much more onerous than those for static DCF analysis. In fact, the only additional inputs needed are the volatility of completed-building prices and the expenditure required to suspend and resume construction. However, although real options analysis uses the same inputs as static DCF analysis (augmented by volatility), we will see that the importance of the various inputs changes when development flexibility is incorporated into the valuation procedure. For example, we find that the market value of a partly-completed project is much more sensitive to the level of the capitalization rate, and much less sensitive to the price of completed buildings, than is suggested by static DCF valuation. Thus, not only does static DCF analysis underestimate the market value of development projects, but it also gives a misleading indication of the determinants of that market value. In short, it tells practitioners to concentrate on building prices when they should perhaps be focussing more on the capitalization rate and price volatility.

The particular example presented in this paper relates to a partly-completed project. The developer has the option to suspend development, an option that many developers have exercised in the 2007–2009 crisis period. In a world of volatile market conditions, the ability to slow down or halt construction, and resume it at a later date, can be extremely valuable. Real options analysis as described in this paper allows practitioners to value projects that have suspended

construction or have the option to do so in the future. Static DCF analysis, in contrast, does not: it implicitly assumes that all future decisions are programmed in advance, which prevents owners from resuming construction when market conditions improve and suspending it when they deteriorate. Other forms of construction flexibility—such as the option to change the scale or scope of a project part way through construction—can be handled using straightforward extensions of the techniques described here.<sup>1</sup>

We begin by describing our hypothetical example and then, in Section 3, we evaluate it using static DCF analysis. Section 4 shows how to set up our valuation model and Section 5 then describes three different approaches to estimating the volatility parameter. The valuation approach is presented in Section 6 and applied to our hypothetical project in Section 7.

## 2 Hypothetical development project

We will illustrate the real options approach using a hypothetical example of a partly completed apartment complex in Las Vegas at the end of 2008.<sup>2</sup> The example involves a 24-story apartment complex that, when completed, will have 650,000 sq ft of rentable space. Similar buildings currently sell for \$160 per sq ft. Construction, which will take 26 months if it is not interrupted, is divided into five distinct stages: construction of the substructure; construction of the lower part of the building's shell; construction of the upper part; the interior fit out of the lower floors; and the interior fit out of the upper floors. The magnitude and timing of the required cash flows are shown in the left-hand panel of Table 1 for the case when the project's construction is not interrupted.<sup>3</sup> We assume that the level of all construction costs is risk free.<sup>4</sup>

Construction can be interrupted between these stages (but not within a stage), but the developer must pay \$10,000 per month while construction is suspended to secure the site, retain the services of key personnel, and so on. The entire project can be abandoned between stages (but, again, not within a stage). The cost of cleaning up the site and shutting the project down almost completely offsets any residual land value, so that the project is worthless if it is abandoned.

We will use 2% as our estimate of the real risk-free interest rate, consistent with the yield on 10-year Treasury inflation-protected securities (TIPS) at the time, and 6% as our estimate of

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<sup>1</sup>See Guthrie (2009b) for many more examples of the types of situations that can be analyzed.

<sup>2</sup>Las Vegas experienced a boom and subsequent bust in its commercial real estate market. For example, according to the Current Employment Statistics produced by the Bureau of Labor Statistics, estimated total employment in Las Vegas in building construction climbed from 11,400 in August 2002 to 19,200 in August 2007, before falling to 12,600 in August 2009.

<sup>3</sup>These cost estimates are broadly in line with those reported by RSMeans for similar projects in Las Vegas during the first quarter of 2009. See <http://www.meanscostworks.com/>.

<sup>4</sup>It is straightforward to incorporate construction-cost risk, although this requires introducing a more complicated valuation model built on a multinomial, rather than binomial, tree.

Table 1: Classifying cash flows needed to construct an apartment complex

Item	Actual project			Model project		
	Amount (\$m)	Date decided (month)	Date incurred (month)	Stage	Duration (periods)	Lump sum cash flow (\$m)
Substructure						
Excavation	0.1	0	0			
Foundations	9.5	0	1			$0.1 + \frac{9.5}{1.02^{1/12}} + \frac{0.2}{1.02^{5/12}}$
Slab on grade	0.2	0	5	I	6	= 9.78
Shell (1st stage)						
Floor construction	3.3	6	6			
Exterior walls	12.5	6	8			
Floor construction	3.3	6	8			
Exterior walls	12.5	6	10			
Floor construction	3.3	6	10			$3.3 + \frac{15.8}{1.02^{2/12}} + \frac{15.8}{1.02^{4/12}} + \frac{12.5}{1.02^{6/12}}$
Exterior walls	12.5	6	12	II	6	= 47.12
Shell (2nd stage)						
Floor construction	3.3	12	12			
Exterior walls	12.5	12	14			
Floor construction	3.3	12	14			
Exterior walls	12.5	12	16			
Roof construction	0.4	12	16	III	6	$3.3 + \frac{15.8}{1.02^{2/12}} + \frac{12.9}{1.02^{4/12}}$ = 31.86
Interior (1st stage)						
Elevators and lifts	9.0	18	18			
Interior fit out	14.0	18	18			$23 + \frac{14}{1.02^{2/12}}$
Interior fit out	14.0	18	20	IV	4	= 36.95
Interior (2nd stage)						
Interior fit out	14.0	22	22			$14 + \frac{14}{1.02^{2/12}}$
Interior fit out	14.0	22	24	V	4	= 27.95

the capitalization rate, consistent with market evidence in the fourth quarter of 2008 (Poutasse, 2009).<sup>5</sup> In order to help us focus on the differences between static DCF analysis and real options analysis, we will ignore taxes.

### 3 Static DCF analysis

Static DCF analysis implicitly assumes that the developer does not exploit any of the flexibility embedded in the project. For the one described in Section 2, we would usually assume that the developer constructs the building as quickly as possible. The estimated market value of the project is the present value of its net cash flows, calculated assuming that construction follows the timetable in the left-hand panel of Table 1. Completing the project is deemed desirable if and only if this net present value is positive.

We could use a single discount rate to calculate the present value of all cash flows. However, in keeping with the notion that discount rates should reflect the systematic risk of the individual cash flows, we will discount capital expenditure using the risk-free interest rate and discount the expected value of the completed building using a rate that reflects the systematic risk of property prices.

If construction has not yet begun, then the present value of the capital expenditure in Table 1 is \$150.50m. To calculate the present value of the building we plan to construct, we could first compound the current market value of an equivalent building ( $0.65 \times 160 = 104$  million dollars) for 26 months using an estimate of the expected growth rate in building prices and then discount this amount back to the present using the risk-adjusted discount rate. It is simpler (and equivalent) to discount the current market value using the capitalization rate, which compensates the owner for the fact that the building does not generate any operating revenue for 26 months. Our capitalization rate of 6% implies a present value of \$91.67m. Static DCF analysis of the project would conclude that construction should not begin.

The approach is similar if the first stage of the exterior shell (only) has already been completed. We should only consider the present value of the capital expenditure required to complete the project, so that sunk expenditure is ignored and the other expenditure is discounted back to month 12, when the first stage of the exterior shell has just been completed. The present value of the remaining capital expenditure is therefore

$$3.3 + \frac{12.5 + 3.3}{1.02^{2/12}} + \frac{12.5 + 0.4}{1.02^{4/12}} + \frac{9.0 + 14.0}{1.02^{6/12}} + \frac{14.0}{1.02^{8/12}} + \frac{14.0}{1.02^{10/12}} + \frac{14.0}{1.02^{12/12}} = 95.95$$

million dollars. Similarly, when calculating the present value of the completed building we need to allow for the shorter time until completion, discounting from month 26 back to month 12.

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<sup>5</sup>The capitalization rate ( $b$ ) is the ratio of net operating income to the market value of the hypothetical completed project. It is related to the expected growth rate in property prices ( $g$ ) and the risk-adjusted discount rate via  $(1 + b)(1 + g) = 1 + \text{RADR}$ .

The present value of the completed building is

$$\frac{104}{1.06^{14/12}} = 97.16$$

million dollars. In this case, static DCF analysis of the project concludes that construction should continue; the implied valuation of the project equals the difference between these two present values, or \$1.22m.

## 4 Modeling development flexibility

There are several key decision points during construction at which the developer must commit to particular future expenditure. For example, the developer must first commit to the expenditure required to excavate the site and complete the building's foundations. The second commitment is to begin construction of the lower part of the building's shell. As shown in Table 1, this commitment can be delayed until the substructure has been completed without construction being interrupted. It can be delayed even longer, although in this case construction would have to be suspended once the substructure is completed. Construction can resume only once the commitment to the second group of cash flows is made. Similarly, the developer can delay committing to the expenditure required to build the top part of the shell until the lower part is completed without construction being interrupted. Longer delays will result in construction being suspended with the shell only partly completed.

We capture this ability to delay committing to some future expenditure by reducing the sequence of cash flows to a smaller set of equivalent lump sum cash flows, each incurred at the time the commitment to incur the relevant future expenditure is made. For example, if the developer decides to begin construction of the substructure at month 0 then the developer commits to spend \$0.1m at month 0, \$9.5m at month 1, and \$0.2m at month 5. This sequence of cash flows is replaced by a single lump sum cash flow of

$$0.1 + \frac{9.5}{1.02^{1/12}} + \frac{0.2}{1.02^{5/12}} = 9.78$$

million dollars that is incurred at the date the decision is made, in this case month 0. Similarly, at month 6 the developer commits to expenditure of \$3.3m at month 6, \$15.8m at month 8, another \$15.8m at month 10, and \$12.5m at month 12. We replace this cash flow stream by lump sum expenditure of

$$3.3 + \frac{15.8}{1.02^{2/12}} + \frac{15.8}{1.02^{4/12}} + \frac{12.5}{1.02^{6/12}} = 47.12$$

million dollars that is incurred at month 6. It is essential that these present values are calculated as at the decision date, not as at month 0. Thus, the cash flow of \$12.5m is discounted six months,

Table 2: Five-stage representation of the apartment project

Stage	Duration (months)	Expenditure (\$m)
I	6	9.78
II	6	47.12
III	6	31.86
IV	4	36.95
V	4	27.95

from month 12 back to month 6, not all the way back to month 0. The results for the remaining cash flows are shown in the right-hand panel of Table 1.

The apartment construction project is therefore reduced to the simpler five-stage project described in Table 2. For each stage the cash flows are collapsed into a single lump sum equal to their present value. The timing of the individual cash flows is important only to the extent that it determines their present value. However, we need to know the overall time required to complete each stage because one stage cannot begin until the previous one has been completed. For example, the developer cannot begin Stage II until at least six months have elapsed after beginning Stage I.

All the information used up to this point is required for a static DCF analysis of the project. The only additional information that we require in order to incorporate the various real options into our project evaluation is the volatility of changes in the sales price of the completed building. The next section shows how to estimate volatility from available market data.

## 5 Estimating volatility

We need to specify the volatility (that is, the standard deviation of the annual rate of return) of the market value of the completed project. The hypothetical sale price of the building currently under construction—if it had just been finished—is clearly unobservable, so that we must rely on other, observable, variables to serve as proxies when estimating volatility. This section describes three possible approaches to estimating the volatility of the completed project’s market value.

The first approach uses information contained in commercial real estate (CRE) indices, which attempt to measure the behavior of diversified portfolios of such assets. Note, however, that we are interested in the behavior of the (hypothetical) market value of a particular building in a particular location. Even those indices that are disaggregated to the level of building classes or geographic locations will feature some degree of diversification. Given that the changes in prices of individual buildings will be less-than-perfectly correlated, this diversification means that the volatility of an index will underestimate the volatility of our building price, perhaps to

Table 3: Estimates of volatility from various CRE price indices

Region	Building	Sample	Standard deviation	
			Quarter	Annual
NCREIF Property Index (NPI)				
National	All	1978:1–2008:4	0.0177	0.0354
National	Apartments	1984:1–2008:4	0.0158	0.0317
Transaction-Based Index (TBI)				
National	All	1984:1–2008:4	0.0387	0.0773
National	Apartments	1994:1–2008:4	0.0408	0.0817
Moodys/REAL CPPI				
National	Apartments	2001:1–2008:3	0.0444	0.0888
Western	Apartments	2001:1–2008:3	0.0426	0.0852

a significant extent.

CRE indices are frequently based on the appraised values of properties in selected portfolios. For example, the National Council of Real Estate Investment Fiduciaries (NCREIF) publishes the NCREIF Property Index (NPI) of investment properties held by pension funds in the United States. The appraisal process tends to smooth out fluctuations in the market value of properties, so that the volatility of indices such as the NPI will tend to underestimate the volatility of the particular diversified portfolio of properties. A better source for our purposes is an index such as the Transaction-Based Index (TBI) produced by the MIT Center for Real Estate. The TBI is calculated using the estimated relationship between transaction prices and property characteristics for properties in the NCREIF portfolio. It will thus do a better job of capturing the actual volatility in market prices than an appraisal-based index. Repeat-transaction indices, such as the Moodys/REAL CPPI produced by the MIT Center for Real Estate and which includes most transactions exceeding \$2.5m, are also beginning to appear.

Table 3 reports volatility estimates derived from the NPI, TPI, and CPPI indices, where all available data have been used for each series. The published index levels have been converted into real terms using the CPI for all urban consumers (US city average) published by the Bureau of Labor Statistics. The first entry in each pair reports the standard deviation of the quarterly change in the logarithm of the price-only index and the second entry gives its annualized counterpart.<sup>6</sup> As expected, the appraisal-based NPI has very low volatility. For the other two indices, volatility estimates range from 0.0773 (the national TPI, with all property types incorporated in a single index) to 0.0888 (the national CPPI for apartments).

Historical data on real estate investment trust (REIT) share price returns provide a second approach to estimating volatility. Because the share prices are true trading prices, the problems

<sup>6</sup>The standard deviation is annualized by multiplying it by the square root of the number of observations per year.

caused by the reliance of CRE indices on appraisals and infrequent transactions are avoided. However, like the CRE indices, REIT prices reflect the values of portfolios of properties. This diversification means that REIT-based volatility is likely to underestimate the volatility for a single building. Downing et al. (2007) have analyzed 18 firm-level REIT returns over the period 1995–2005. They obtain an average de-levered volatility estimate for multi-family properties of 0.107, with a standard deviation of 0.031.

The third approach that we consider uses observed prices of financial securities with values that are sensitive to the volatility of property prices. Given a suitable valuation model, the so-called “implied volatility” is the level of volatility that sets the model’s predicted price equal to the observed price.<sup>7</sup> Implied volatility is a forward-looking estimate, being based on the market’s expectations of future volatility. In contrast, because they are based on historical data, the first two approaches described above are backward-looking. Downing et al. (2007) calculate the level of volatility implied by each of 4,032 CRE loans involving multi-family properties issued during the period 1996–2005. The implied volatilities that they calculate have a mean of 0.198 and a standard deviation of 0.079.<sup>8</sup> Using loans on individual properties has the advantage that the implied volatility reflects the volatility of the price of a single property: the diversification present in CRE indices and REIT returns is absent.

These three approaches produce volatility estimates ranging from approximately 0.08 (CRE indices based on property portfolios) to 0.20 and above (volatility implied by loans on individual properties). Since we require an estimate of the volatility for a single property, we set volatility equal to  $\sigma = 0.20$ .

## 6 Using real options analysis to evaluate the project

We carry out our analysis of this problem using the framework of a binomial tree, which we use to describe the behavior of the hypothetical price of a completed project. The  $T$ -year lifetime of the development rights is divided into a sequence of  $N$  periods, each representing  $\Delta t = T/N$  years. During each one of these periods the hypothetical price of a completed project changes by a constant factor of either  $U = e^{\sigma\sqrt{\Delta t}} > 1$  or  $D = e^{-\sigma\sqrt{\Delta t}} < 1$ . The starting value, denoted  $X_0$ , is the hypothetical market value the project would have today if it had just been completed. In our example,  $X_0 = 104$  million. By the time we get to date  $n$ , when there will have been

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<sup>7</sup>This approach is often used in derivative pricing applications, but it can also be used to estimate other parameters of interest. For example, Boyle et al. (2009) use it to estimate the parameters needed to calculate “fair-value” ground rentals.

<sup>8</sup>Older studies obtain similar values. For example, Titman and Torous (1989) calculate an implied volatility of building prices of 0.1550 from quoted rates on commercial mortgages during the period 1985–1987; Holland et al. (2000) use mortgage rates reported by the American Council of Life Insurers during the period 1979–1992 to obtain estimates ranging from 0.15 to 0.25.

some random number  $i$  down moves and  $n - i$  up moves, the hypothetical market value will have changed to  $V_0(i, n) = X_0 D^i U^{n-i}$ . The top left-hand corner of the resulting binomial tree is shown in the first panel in Table 4, with each row corresponding to a different number ( $i$ ) of down moves and each column ( $n$ ) to a different date.<sup>9</sup> This tree completely describes our uncertainty about future property prices. For example, six periods from now, the hypothetical price of the completed project could be as low as  $X_0 D^6$ , as high as  $X_0 U^6$ , or somewhere in between. The possible prices comprise the column labeled by  $n = 6$  in the first panel in Table 4.

The binomial tree for  $V_0$  is the basis of our valuation model. No matter what stage of construction the project is at, its market value will depend on the hypothetical price of the project if it had just been completed. Thus, we use  $V_m(i, n)$  to denote the market value of the project rights if  $m$  construction stages remain and a completed project could—hypothetically—currently be sold for  $V_0(i, n)$ . We use a separate table for each of the five values of  $m$ , with the entry in cell  $(i, n)$  of the  $m$ th of these tables giving the level of  $V_m(i, n)$ .

In order to understand how our modeling approach works we must understand how we move through the trees as construction progresses and market conditions evolve. We start at node  $(0, 0)$  of the bottom panel, which will give the estimated current market value of the project rights if construction is yet to begin. If the developer decides to wait before beginning construction then—because no construction is going to occur in the meantime—we stay in the same panel, but move one column to the right each period, moving down one row each time a down move occurs and otherwise staying in the current row. In contrast, if the developer decides to construct the project’s first stage, we jump up to the panel directly above the bottom one, moving as many columns to the right as it takes periods to complete that stage. Where exactly we emerge in that column depends on how the market has behaved during construction. If every move during construction of the first stage has been up then we will emerge in the same row, but for each down move that occurred during this period we will emerge one row lower. We move through the panels like this, moving rightwards through each panel and occasionally jumping up to the next one. Provided we do not permanently abandon the project along the way, we will eventually arrive at either the top panel, indicating that the project’s construction has been completed, or the final column of one of the other panels, indicating that the development rights have expired before the project could be completed.

Now that we know how to navigate through the panels, we turn to the task of constructing them. All the required calculations can be carried out in a spreadsheet containing six tables, one for each of the project market values ( $V_m$ ). The first table extends out as far as the development’s expiry date—so that the tables can be large—but each successive table is smaller than the one

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<sup>9</sup>Because there cannot be more down moves than periods, the part of the table below the diagonal does not need to be filled in.

Table 4: Overview of solution technique

$V_0(i, n)$	0	1	2	3	4	5	6	...	
0	$V_0(0, 0)$	$V_0(0, 1)$	$V_0(0, 2)$	$V_0(0, 3)$	$V_0(0, 4)$	$V_0(0, 5)$	$V_0(0, 6)$	...	
1		$V_0(1, 1)$	$V_0(1, 2)$	$V_0(1, 3)$	$V_0(1, 4)$	$V_0(1, 5)$	$V_0(1, 6)$	...	
2			$V_0(2, 2)$	$V_0(2, 3)$	$V_0(2, 4)$	$V_0(2, 5)$	$V_0(2, 6)$	...	
3				$V_0(3, 3)$	$V_0(3, 4)$	$V_0(3, 5)$	$V_0(3, 6)$	...	← Step 1: Fill in to date $N$ using $V_0(i, n) = X_0 D^i U^{n-i}$
4					$V_0(4, 4)$	$V_0(4, 5)$	$V_0(4, 6)$	...	
5						$V_0(5, 5)$	$V_0(5, 6)$	...	
6							$V_0(6, 6)$	...	
$V_1(i, n)$	0	1	2	3	4	5	6	...	
0	$V_1(0, 0)$	$V_1(0, 1)$	$V_1(0, 2)$	$V_1(0, 3)$	$V_1(0, 4)$	$V_1(0, 5)$	$V_1(0, 6)$	...	
1		$V_1(1, 1)$	$V_1(1, 2)$	$V_1(1, 3)$	$V_1(1, 4)$	$V_1(1, 5)$	$V_1(1, 6)$	...	← Step 2a: Fill in date $N - N_1 + 1$ with zeros
2			$V_1(2, 2)$	$V_1(2, 3)$	$V_1(2, 4)$	$V_1(2, 5)$	$V_1(2, 6)$	...	
3				$V_1(3, 3)$	$V_1(3, 4)$	$V_1(3, 5)$	$V_1(3, 6)$	...	
4					$V_1(4, 4)$	$V_1(4, 5)$	$V_1(4, 6)$	...	
5						$V_1(5, 5)$	$V_1(5, 6)$	...	← Step 2b: Fill in earlier col's using eq. (1) with $m = 1$
6							$V_1(6, 6)$	...	
$V_2(i, n)$	0	1	2	3	4	5	6	...	
0	$V_2(0, 0)$	$V_2(0, 1)$	$V_2(0, 2)$	$V_2(0, 3)$	$V_2(0, 4)$	$V_2(0, 5)$	$V_2(0, 6)$	...	
1		$V_2(1, 1)$	$V_2(1, 2)$	$V_2(1, 3)$	$V_2(1, 4)$	$V_2(1, 5)$	$V_2(1, 6)$	...	← Step 3a: Fill in date $N - N_1 - N_2 + 1$ with zeros
2			$V_2(2, 2)$	$V_2(2, 3)$	$V_2(2, 4)$	$V_2(2, 5)$	$V_2(2, 6)$	...	
3				$V_2(3, 3)$	$V_2(3, 4)$	$V_2(3, 5)$	$V_2(3, 6)$	...	
4					$V_2(4, 4)$	$V_2(4, 5)$	$V_2(4, 6)$	...	
5						$V_2(5, 5)$	$V_2(5, 6)$	...	← Step 3b: Fill in earlier col's using eq. (1) with $m = 2$
6							$V_2(6, 6)$	...	
$V_3(i, n)$	0	1	2	3	4	5	6	...	
0	$V_3(0, 0)$	$V_3(0, 1)$	$V_3(0, 2)$	$V_3(0, 3)$	$V_3(0, 4)$	$V_3(0, 5)$	$V_3(0, 6)$	...	
1		$V_3(1, 1)$	$V_3(1, 2)$	$V_3(1, 3)$	$V_3(1, 4)$	$V_3(1, 5)$	$V_3(1, 6)$	...	← Step 4a: Fill in date $N - N_1 - N_2 - N_3 + 1$ with zeros
2			$V_3(2, 2)$	$V_3(2, 3)$	$V_3(2, 4)$	$V_3(2, 5)$	$V_3(2, 6)$	...	
3				$V_3(3, 3)$	$V_3(3, 4)$	$V_3(3, 5)$	$V_3(3, 6)$	...	
4					$V_3(4, 4)$	$V_3(4, 5)$	$V_3(4, 6)$	...	
5						$V_3(5, 5)$	$V_3(5, 6)$	...	← Step 4b: Fill in earlier col's using eq. (1) with $m = 3$
6							$V_3(6, 6)$	...	
$V_4(i, n)$	0	1	2	3	4	5	6	...	
0	$V_4(0, 0)$	$V_4(0, 1)$	$V_4(0, 2)$	$V_4(0, 3)$	$V_4(0, 4)$	$V_4(0, 5)$	$V_4(0, 6)$	...	
1		$V_4(1, 1)$	$V_4(1, 2)$	$V_4(1, 3)$	$V_4(1, 4)$	$V_4(1, 5)$	$V_4(1, 6)$	...	← Step 5a: Fill in date $N - N_1 - \dots - N_4 + 1$ with zeros
2			$V_4(2, 2)$	$V_4(2, 3)$	$V_4(2, 4)$	$V_4(2, 5)$	$V_4(2, 6)$	...	
3				$V_4(3, 3)$	$V_4(3, 4)$	$V_4(3, 5)$	$V_4(3, 6)$	...	
4					$V_4(4, 4)$	$V_4(4, 5)$	$V_4(4, 6)$	...	
5						$V_4(5, 5)$	$V_4(5, 6)$	...	← Step 5b: Fill in earlier col's using eq. (1) with $m = 4$
6							$V_4(6, 6)$	...	
$V_5(i, n)$	0	1	2	3	4	5	6	...	
0	$V_5(0, 0)$	$V_5(0, 1)$	$V_5(0, 2)$	$V_5(0, 3)$	$V_5(0, 4)$	$V_5(0, 5)$	$V_5(0, 6)$	...	
1		$V_5(1, 1)$	$V_5(1, 2)$	$V_5(1, 3)$	$V_5(1, 4)$	$V_5(1, 5)$	$V_5(1, 6)$	...	← Step 6a: Fill in date $N - N_1 - \dots - N_5 + 1$ with zeros
2			$V_5(2, 2)$	$V_5(2, 3)$	$V_5(2, 4)$	$V_5(2, 5)$	$V_5(2, 6)$	...	
3				$V_5(3, 3)$	$V_5(3, 4)$	$V_5(3, 5)$	$V_5(3, 6)$	...	
4					$V_5(4, 4)$	$V_5(4, 5)$	$V_5(4, 6)$	...	
5						$V_5(5, 5)$	$V_5(5, 6)$	...	← Step 6b: Fill in earlier col's using eq. (1) with $m = 5$
6							$V_5(6, 6)$	...	

before it.

Construction of the first panel in Table 4 is simplest since the entry at node  $(i, n)$  can be calculated directly as  $V_0(i, n) = X_0 D^i U^{n-i}$ . However, construction of the other panels is more complicated. We saw above that the system evolves starting at the bottom panel and then jumping up one panel at a time, and moving from left-to-right within each panel. This direction needs to be reversed when constructing the tables. That is, we actually work backwards through time, starting with the table when there is just one construction stage remaining and filling it in from right-to-left, before moving to the table when there are two stages remaining, and repeating the procedure. We continue like this, until we eventually calculate the left-most cell of the bottom table.

For each table, we begin by filling in the final column—corresponding to immediately after the latest date on which that stage of construction can begin—with zeros. Suppose that the next construction stage takes  $N_m$  periods to complete if there are  $m$  separate stages remaining. Then if the developer has not begun the final stage on or before date  $N - N_1$ , the rights are worthless (since the project cannot be completed before the rights expire) and  $V_1(i, N - N_1 + 1) = 0$  for all entries in that column. Similarly, if the developer has not begun the penultimate stage on or before date  $N - N_1 - N_2$  then the rights are also worthless, so that we make  $V_2(i, N - N_1 - N_2 + 1) = 0$  for all entries in that column. We then fill in the remainder of the table, working from right to left one column at a time, using the method that we now describe.

Consider what happens at an arbitrary node,  $(i, n)$ , when there are still  $m$  stages remaining to be completed. The developer has three choices: abandon the project permanently; suspend construction (possibly temporarily); or construct the next stage immediately. The developer will choose an action that maximizes the market value of the project rights, so that  $V_m(i, n)$  equals the maximum of the payoffs for these three actions. That is,

$$V_m(i, n) = \max\{\text{payoff}_{\text{abandon}}(i, n), \text{payoff}_{\text{suspend}}(i, n), \text{payoff}_{\text{develop}}(i, n)\}. \quad (1)$$

This equation tells us how much the development rights are worth at this node and—by checking which of the three payoffs is largest—indicates the optimal development policy. However, before we can evaluate equation (1) we need to know how to calculate the three payoffs that it contains.

**Abandonment** The first payoff occurs when the developer abandons the project and receives the salvage value  $A$ , for a payoff of

$$\text{payoff}_{\text{abandon}}(i, n) = A.$$

**Suspension** Alternatively, the developer can pay  $H$  and temporarily suspend construction, holding the project in its current state for one period. We already know that after one period has passed we will be in the next column of the same table, in the same row if the

move is up and in the row below if it is down. That is, the project rights will be worth  $V_m(i, n + 1)$  if an up move occurs and  $V_m(i + 1, n + 1)$  if the move is down. In traditional DCF analysis, we would calculate the market value of these possibilities by calculating the expected value and then discounting it back one period using a risk-adjusted discount rate. Real options analysis achieves exactly the same market-value estimate by altering the expected cash flow and then using the risk-free interest rate to do the discounting. Specifically, the payoff from suspension is

$$\text{payoff}_{\text{suspend}}(i, n) = -H + (1 + r)^{-\Delta t} \left( \pi_u V_m(i, n + 1) + (1 - \pi_u) V_m(i + 1, n + 1) \right),$$

where

$$\pi_u = \frac{\left( \frac{1+r}{1+b} \right)^{\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}},$$

$r$  is the risk-free interest rate,  $b$  is the capitalization rate, and  $\sigma$  is the volatility of the value of the completed project.<sup>10</sup> The term in large brackets is the expected value of the rights after one period (calculated using the “risk-neutral probability”  $\pi_u$ ), which is then discounted back one period using the risk-free interest rate.

**Construction** The third payoff is the most complicated to evaluate. If the developer decides to construct the next stage of the project, then he commits to a series of cash outflows with present value  $I_m$ . Since this stage takes  $N_m$  periods to complete, there are many possible outcomes to consider, one for each number of down moves that occur during construction. For example, if there are  $j$  down moves (and so  $N_m - j$  up moves), the project rights will be worth  $V_{m-1}(i + j, n + N_m)$  when this stage of construction is completed. The subscript indicates that there will be just  $m - 1$  stages remaining, the row number indicates that there have been  $i + j$  down moves since date 0 in total, while the column number indicates that we will be at date  $n + N_m$ . When we take all possibilities into account, we find that the payoff from completing the project’s next stage is

$$\text{payoff}_{\text{develop}}(i, n) = -I_m + (1 + r)^{-N_m \Delta t} \sum_{j=0}^{N_m} \phi(j, N_m) \cdot V_{m-1}(i + j, n + N_m),$$

where  $\phi(j, N_m)$  is the probability that exactly  $j$  of the next  $N_m$  moves are down, where each move is down with probability  $1 - \pi_u$ .<sup>11</sup> The first term is the present value of the required capital expenditure, while the second term is the expected payoff (calculated using the risk-neutral probability), discounted back to the present using the risk-free interest rate.

<sup>10</sup>Derivations of this and other results can be found in Chapter 3 of Guthrie (2009b).

<sup>11</sup>It can be calculated in EXCEL using  $\phi(j, N_m) = \text{BINOMDIST}(j, N_m, 1 - \pi_u, \text{FALSE})$ . Alternatively, it can be calculated directly using

$$\phi(j, N_m) = \frac{N_m!}{j!(N_m - j)!} \pi_u^{N_m - j} (1 - \pi_u)^j.$$

Table 5: Summary of solution technique

Step	Instruction
1	Fill in the tree for $V_0$ out as far as date $N$ using $V_0(i, n) = X_0 D^i U^{n-i}$
2a	Set all values of $V_1$ at date $N - N_1 + 1$ equal to zero
2b	Moving one column left at a time, fill in the remaining columns of the tree for $V_1$ using equation (1) with $m = 1$
3a	Set all values of $V_2$ at date $N - N_1 - N_2 + 1$ equal to zero
3b	Fill in the remaining columns of the tree for $V_2$ using equation (1) with $m = 2$
4a	Set all values of $V_3$ at date $N - N_1 - N_2 - N_3 + 1$ equal to zero
4b	Fill in the remaining columns of the tree for $V_3$ using equation (1) with $m = 3$
5a	Set all values of $V_4$ at date $N - N_1 - N_2 - N_3 - N_4 + 1$ equal to zero
5b	Fill in the remaining columns of the tree for $V_4$ using equation (1) with $m = 4$
6a	Set all values of $V_5$ at date $N - N_1 - N_2 - N_3 - N_4 - N_5 + 1$ equal to zero
6b	Fill in the remaining columns of the tree for $V_5$ using equation (1) with $m = 5$

Construction of the suspension and construction payoffs for the special case where  $m = 2$ ,  $i = 1$ , and  $n = 1$  is illustrated in Table 4. We are currently at the node indicated by the boxed cell, which shows  $V_2(1, 1)$ . The payoff from suspension depends on the two shaded cells in the same table since the value of the development rights changes to either  $V_2(1, 2)$  or  $V_2(2, 2)$  after one period elapses. We take the weighted average of these two entries (where the weights are the risk-neutral probabilities), discount back one period using the risk-free interest rate, and then subtract one period's worth of suspension expenditure. In contrast, the payoff from developing this stage of the project depends on the shaded cells in the second table. Assuming monthly time steps, construction will take four periods, so that the rights will be worth one of  $V_1(1, 5), \dots, V_1(5, 5)$  as soon as construction of the next stage is complete. To calculate the development payoff, we take the weighted average of these five entries (where the weights are the risk-neutral probabilities given by the  $\phi$ s), discount back four periods using the risk-free interest rate, and then subtract the lump sum development expenditure.

The entire solution process is summarized in Table 5.

## 7 Results

Our parameter values are reported in Table 6. Using monthly time steps (that is,  $\Delta t = 1/12$ ) implies up and down moves of  $U = 1.0594$  and  $D = 0.9439$ , respectively, and a risk-neutral probability of an up move of  $\pi_u = 0.4579$ .

Table 7 shows an extract from the spreadsheet constructed using the approach described in Section 6, corresponding to the first six months of each table. The entry in each cell gives the market value of the development rights for the indicated number of remaining stages, with

Table 6: Parameter values

Parameter	Symbol	Value
Needed for static DCF analysis		
Current sale price	$X_0$	104
Capitalization rate	$b$	0.06
Risk-free interest rate	$r$	0.02
Capital expenditure	$I_5$	9.78
	$I_4$	47.12
	$I_3$	31.86
	$I_2$	36.95
	$I_1$	27.95
Construction times*	$N_5$	6
	$N_4$	6
	$N_3$	6
	$N_2$	4
	$N_1$	4
Not needed for static DCF analysis		
Price volatility	$\sigma$	0.20
Salvage payoff	$A$	0
Suspension expenditure*	$H$	0.01
Lifetime of development rights*	$N$	120

\*Assumes monthly time steps

shaded cells indicating situations when the next stage should be constructed immediately. For example, if the project had just been completed it would be worth \$104m (the top left-hand entry of the first table). It would be worth just \$74.05m if a single stage remained (the top left-hand entry of the second table), in which case the remaining stage should be built immediately. Similarly, the project would be worth \$35.31m if two stages remain and \$1.67m if no construction at all has occurred.

Table 8 compares the results of our real options analysis with those of static DCF analysis, as described in Section 3. The first two entries in each row give the market value of the development rights if the building price is \$160 per sq ft, calculated using the static DCF approach described in Section 3, and the break-even level of the building price (that is, the level of the building price that makes the present value of the remaining capital expenditure equal the present value of the completed building, both calculated assuming that development is not interrupted). The next two entries report analogous information calculated using the real options approach described in Section 6. That is, the market values are the values of  $V_m(0, 0)$  calculated in the spreadsheet summarized in Table 7 and the development thresholds are the lowest building prices for which

Table 7: Spreadsheet extract

$V_0(i, n)$	0	1	2	3	4	5	6	...
0	104.00	110.18	116.73	123.67	131.02	138.80	147.05	...
1		98.17	104.00	110.18	116.73	123.67	131.02	...
2			92.66	98.17	104.00	110.18	116.73	...
3				87.46	92.66	98.17	104.00	...
4					82.55	87.46	92.66	...
5						77.92	82.55	...
6							73.55	...
$V_1(i, n)$	0	1	2	3	4	5	6	...
0	74.05	80.11	86.53	93.33	100.54	108.18	116.27	...
1		68.32	74.05	80.11	86.53	93.33	100.54	...
2			62.92	68.32	74.05	80.11	86.53	...
3				57.82	62.92	68.32	74.05	...
4					53.01	57.82	62.92	...
5						48.47	53.01	...
6							44.18	...
$V_2(i, n)$	0	1	2	3	4	5	6	...
0	35.31	41.26	47.56	54.23	61.30	68.79	76.73	...
1		29.70	35.31	41.26	47.56	54.23	61.30	...
2			24.40	29.70	35.31	41.26	47.56	...
3				19.80	24.40	29.70	35.31	...
4					16.00	19.80	24.40	...
5						12.85	15.99	...
6							10.26	...
$V_3(i, n)$	0	1	2	3	4	5	6	...
0	11.43	14.33	17.87	22.19	27.42	33.77	41.44	...
1		9.03	11.40	14.30	17.85	22.17	27.41	...
2			7.07	9.00	11.37	14.28	17.83	...
3				5.48	7.04	8.97	11.34	...
4					4.20	5.45	7.01	...
5						3.17	4.17	...
6							2.35	...
$V_4(i, n)$	0	1	2	3	4	5	6	...
0	2.55	3.41	4.51	5.89	7.62	9.78	12.44	...
1		1.84	2.52	3.38	4.47	5.85	7.57	...
2			1.30	1.81	2.48	3.34	4.42	...
3				0.88	1.27	1.79	2.45	...
4					0.58	0.86	1.25	...
5						0.35	0.56	...
6							0.20	...
$V_5(i, n)$	0	1	2	3	4	5	6	...
0	1.67	2.31	3.12	4.16	5.48	7.14	9.21	...
1		1.16	1.65	2.27	3.08	4.12	5.43	...
2			0.78	1.14	1.62	2.24	3.04	...
3				0.50	0.76	1.11	1.58	...
4					0.29	0.48	0.74	...
5						0.15	0.28	...
6							0.07	...

Table 8: Comparison of static DCF and real options analysis

No. stages remaining $m$	Static DCF analysis		Real options analysis	
	Market value (\$m)	Price threshold (\$ per sq ft)	Market value (\$m)	Price threshold (\$ per sq ft)
5	-58.84	263	1.67	341
4	-47.75	241	2.55	328
3	1.22	158	11.43	215
2	35.31	104	35.31	140
1	74.05	44	74.05	59

it is optimal to immediately construct the next stage of the project at date 0.<sup>12</sup>

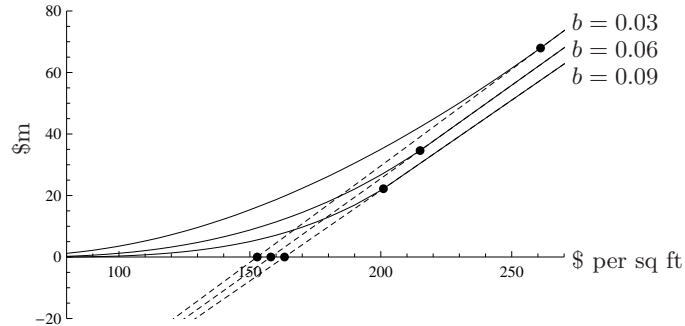
If construction of the project has not begun, then static DCF analysis severely underestimates the market value of the project, giving it a negative value when—due to the option to delay construction until market conditions improve—the project rights actually have a positive value. Moreover, it is optimal to begin development only when the building price is much higher than the level predicted by static DCF analysis (\$341 per sq ft rather than \$263 per sq ft). The higher building price is needed to compensate the developer for the loss of the delay option that is destroyed when construction begins. This is also the case when the first stage has been completed.

With three stages remaining, static DCF analysis would incorrectly suggest that construction should be resumed immediately, since the current building price of \$160 per sq ft exceeds the static-DCF threshold. However, Table 8 shows that development should actually be delayed until the building price rises to at least \$215 per sq ft. It also shows that the development rights are worth almost ten times the value predicted by static DCF analysis. Thus, using static DCF analysis to evaluate projects for which the development option is at-the-money (that is, the building price is currently close to the break-even threshold) undervalues the projects and leads to too much development.

When there are only one or two stages remaining, static DCF analysis performs well for both valuation and decision making, because in this case the value of being able to delay exercising the development option is very low. As Table 8 shows, in this case the current building price of \$160 per sq ft is substantially above the optimal development threshold, so that the development option is deep in-the-money. This behavior suggests that when analyzing a project, the flexibility to suspend development in the early stages—when the development threshold is high and the development option is less likely to be deep in-the-money—is more valuable than flexibility to delay completing the final few stages. Thus, when simplifying a project into a small number

<sup>12</sup>When using EXCEL, the optimal threshold can be found by using the Solver function to find the value of  $X_0$  for which  $\text{payoff}_{\text{develop}}(0, 0) = \text{payoff}_{\text{wait}}(0, 0)$ .

Figure 1: Sensitivity analysis with three stages remaining: building price and capitalization rate



of stages as in Section 4 it is important to focus on separating out various stages early in the development process; later stages can be grouped together into a small number of stages without greatly compromising our analysis.

The approach described in this paper assumes that the underlying parameters are known with certainty. However, in practice, most parameters are estimated with uncertainty. One possible response to this parameter uncertainty is to carry out sensitivity analysis, starting with a distribution of possible parameter values and finishing with the implied distribution of project values. Because the approach in this paper can be implemented within a spreadsheet, construction of the distribution can be carried out in the same way as for static DCF analysis. Rather than repeat that here, we restrict ourselves to examining the effect of changes in three important parameters—building price, capitalization rate, and volatility—on the market value of the development rights.

Figure 1 plots the market value (in \$m) of the development rights when three stages remain, as a function of the current building price (in \$ per sq ft). The three solid curves show the real-options valuations corresponding to different capitalization rates, 3% (top), 6% (middle), and 9% (bottom); the dashed curves show the corresponding static DCF valuations. All other parameters take the values in Table 6. For each combination of building price and capitalization rate, the height of the curve equals  $V_3(0, 0)$ . The dot on each curve indicates the lowest level of the building price at which it is optimal to immediately begin construction of the third-to-last stage of the project. Real options analysis yields price thresholds ranging from \$201 per sq ft when the capitalization rate is 9% to \$261 per sq ft when it is 3%; static DCF analysis gives respective thresholds of \$153 per sq ft and \$163 per sq ft. Two points stand out. Firstly, static DCF analysis gives development thresholds that are far too low. Secondly, it gives a misleading view of the sensitivity of the optimal development threshold to the level of the capitalization rate.

Figure 2: Sensitivity analysis with three stages remaining: building price and volatility

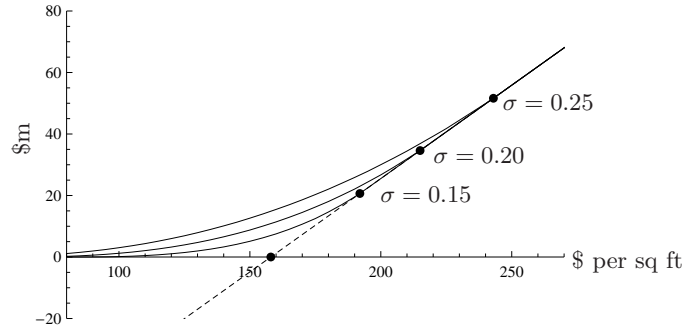


Figure 2 repeats the format of the preceding one, but this time the three solid curves show the real-options valuations corresponding to different volatilities, 0.15 (bottom), 0.20 (middle), and 0.25 (top). Because volatility does not affect static DCF analysis, there is just one dashed curve showing the corresponding static DCF valuations. All other parameters take the values in Table 6. Real options analysis yields price thresholds ranging from \$192 per sq ft when volatility is 0.15 to \$243 per sq ft when it is 0.25. Both the value of the project rights and the optimal development threshold are sensitive to the level of price volatility.

The relative importance of these three parameters can be measured by calculating the change in each parameter—while holding all other parameters at their baseline levels—required to increase the project rights’ market value by 10% (that is, from \$11.43m to \$12.56m). Numerical search routines (such as Solver in EXCEL) can be used to calculate that the building price needs to rise by 2.4% (from \$160.00 per sq ft to \$163.79 per sq ft), or price volatility needs to rise by 7% (from 0.200 to 0.214), or the capitalization rate needs to fall by 10% (from 0.060 to 0.054). In contrast, the static DCF valuation of the project rights increases by the same (absolute) amount of \$1.14m if the building price rises by 1.2% (to \$161.88 per sq ft) or the capitalization rate falls by 18% (to 0.049); changes in price volatility have no effect on the static DCF valuation. Thus, static DCF valuation overstates the importance to valuation of the building price and understates the importance of the capitalization rate; it entirely misses the importance of price volatility.

## 8 Conclusion

This paper has demonstrated a straightforward method for incorporating construction flexibility in the evaluation of commercial real estate developments. All the calculations can be performed in a spreadsheet and only one parameter—the volatility of the price of a completed project—needs to be estimated in addition to those required for static DCF analysis. The key step is to

allocate individual items of capital expenditure to distinct stages of development. The method allows the developer to suspend construction between stages, thus capturing much of the value of the real options embedded in typical development projects. Reducing the project to a small number of stages means that we need to build only a few tables in the spreadsheet: one for the market value of the completed project and one for each construction stage.

To make the calculations as simple as possible, we suppose that the developer's decision concerns only the timing of construction. Other possibilities, such as varying the rate of construction according to market conditions and deciding to change the project's design mid-construction, can also be analyzed using minor modifications of the techniques described in this paper (Guthrie, 2009b, Chapter 9). Other extensions are possible, but substantially complicate the analysis. For example, it is possible to incorporate uncertainty surrounding the technological requirements of a project. A closely-related issue is the uncertainty surrounding future levels of some parameters. That is, even if we know the level of the interest rate now, we do not know its value one year from now with certainty. Sensitivity analysis can be used here, but—because it varies the level of a parameter while assuming that it will be constant at this level—it does not capture all of the consequences of parameter volatility. A better approach is to explicitly include the relevant parameter as an additional state variable. Again, this can be incorporated, but only if we are willing to use multinomial trees rather than the simple binomial trees used in this paper.

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